

QCD corrections to the production of $t\bar{t}\gamma$ at the ILC

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Abstract

A precise calculation of the top quark pair production associated with a hard photon is essential for testing the electroweak property of the top quark in the Standard Model (SM). We investigate the one-loop QCD corrections to the process $e^+e^- \rightarrow t\bar{t}\gamma$ at the International Linear Collider (ILC), and find that the K -factor can be as large as 1.238 (1.105, 1.060) for a center-of-mass energy $\sqrt{s} = 500$ (800, 1500) GeV. The transverse momentum distributions of the top quark and photon are respectively shown at leading order (LO) and next-to-leading order (NLO). Due to the asymmetric rapidity distribution of the top (anti-top) quark, we also study the top quark forward-backward asymmetry (A_{FB}^t) in $t\bar{t}\gamma$ production at NLO, which is found to be 45.82 (55.25, 55.89)% for $\sqrt{s} = 500$ (800, 1500) GeV.

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I. INTRODUCTION

The Standard Model has been well testified by various experiments [1], except that the Higgs scalar is still left as a missing piece. Therefore, testing the mechanism of electroweak symmetry breaking (EWSB) [2] is now an urgent task for particle physics. The top quark, which was discovered at the Tevatron in 1995 [3], is distinguished for its large mass and short lifetime [4]. Since it is free from the QCD confinement, the top quark productions and decays are much cleaner than the light quarks [5]. Thus, the top quark is speculated to be a sensitive probe for the EWSB and new physics [6]. So far, except for the forward-backward asymmetry in $t\bar{t}$ production at the Tevatron [7], most of the measurements on the top quark are consistent with the Standard Model (SM) predictions, such as the cross sections of the $t\bar{t}$ production and the single top production [8].

Very recently, the CDF Collaboration has reported its observation of $t\bar{t}\gamma$ events with a luminosity of 6.0 fb^{-1} [9]. However, the small statistics still limits the precision study of the gauge coupling of $t\bar{t}\gamma$. In addition, since the initial photon radiation severely affects the sensitivity of $t\bar{t}\gamma$ production to anomalous top quark couplings [10], the SM electric charge of the top quark has not been measured directly up to now. Although the dominant contribution to the $t\bar{t}\gamma$ production comes from the gluon fusion at the LHC, it is challenging to determine the top charge by measuring the cross section ratio $\sigma(t\bar{t}\gamma)/\sigma(t\bar{t})$ [11] due to the huge QCD backgrounds. The LO and NLO QCD calculations of the process $pp(\bar{p}) \rightarrow t\bar{t}\gamma$ have been recently carried out at hadron colliders in Ref.[12].

In contrast, as a clean top quark factory, the ILC will allow for a precision test for the top quark property [13]. Since the gauge coupling of $t\bar{t}\gamma$ is sensitive to new physics, it has been studied intensively in a model independent way [14]. Some new physics models can also affect the $t\bar{t}\gamma$ coupling sizably, such as the Little Higgs model [15]. It is also found that in the supersymmetric and multi-Higgs models [16] a sizable top quark electric(weak) dipole momentum can be induced by the non-standard CP violating interactions. At the ILC, due to a high luminosity, such anomalous couplings of the top quark can be measured at the one percent level, which is much better than that at the LHC [13]. Therefore, the high order calculations for the top quark processes at the ILC are needed to meet the experimental precision. The QCD and electroweak corrections to the process $e^+e^-, \gamma\gamma \rightarrow t\bar{t}$ have been studied in Ref.[17]. The CP violation effects induced at loop level in the top pair production

through (un)polarized e^+e^- annihilation [18] or photon fusion [19] have been investigated in some extensions of the SM. In Ref.[20], the authors have studied the QCD corrections to the $b\bar{b}\gamma$ production at LEP1. In this paper, we calculate the one-loop QCD corrections to the process $e^+e^- \rightarrow t\bar{t}\gamma$ at the ILC.

This work is organized as follows. In section II, a brief description for the NLO QCD calculations is given. The discussions and numerical studies are presented in section III. Finally, the conclusions are drawn in section V.

II. A DESCRIPTION OF ANALYTICAL CALCULATIONS

In our calculations, the NLO QCD corrections ($\Delta\sigma_{QCD}$) are divided into two parts: the virtual corrections ($\Delta\sigma_{vir}$) and the real gluon radiation corrections ($\Delta\sigma_{real}$). We adopt the dimensional regularization to isolate all the ultraviolet divergences (UV) in the one-loop amplitudes generated by FeynArts [21] and remove them with the on-mass-shell renormalization scheme [22]. The FormCalc-6.1 [23] and LoopTools-2.5 [24] are employed to simplify the amplitudes and to perform the numerical calculations respectively.

On the other hand, the infrared (IR) divergences arising from the contributions of virtual gluon exchange in loops are still left. According to the Kinoshita-Lee-Nauenberg (KLN) theorem [25], these IR divergences will be canceled by the real gluon bremsstrahlung corrections in the soft gluon limit. We denote the momentums of initial and final states for the real gluon emission process as follows:

$$e^+(p_1) + e^-(p_2) \rightarrow t(k_1) + \bar{t}(k_2) + \gamma(k_3) + g(k). \quad (1)$$

We take the phase-space-slicing method to isolate the IR singularity and divide the real corrections into the hard and soft parts by the energy of the emitted gluon in the calculations [26, 27]:

$$\Delta\sigma_{real} = \Delta\sigma_{soft} + \Delta\sigma_{hard}. \quad (2)$$

In the soft gluon approximation [28], we can obtain the soft part of the cross sections by the following equation:

$$d\Delta\sigma_{soft} = d\sigma_0 \frac{\alpha_s C_F}{2\pi^2} \int_{E_g \leq \Delta E_g} \frac{d^3\vec{k}}{2E_g} \left(\frac{k_1}{k_1 \cdot k} - \frac{k_2}{k_2 \cdot k} \right)^2, \quad (3)$$

where $C_F = \frac{4}{3}$, $E_g = \sqrt{|\vec{k}|^2 + m_g^2}$ and we give a fictitious mass m_g to the gluon to eliminate the IR divergence. Note that this dependence on the non-physical mass will be exactly canceled by the virtual corrections which are also evaluated with a non-zero gluon mass. ΔE_g is the energy cutoff of the soft gluon and we require $E_g \leq \Delta E_g \ll \sqrt{s}/2$. The hard gluon ($E_g \geq \Delta E_g$) radiation corrections, which are insensitive to the small gluon mass, can be directly evaluated by the numerical Monte Carlo method [29].

Finally, the finite total cross section of the process $e^+e^- \rightarrow t\bar{t}\gamma$ including the LO (σ_0) and NLO QCD corrections ($\Delta\sigma_{QCD}$) can be expressed as

$$\sigma_{tot} = \sigma_0 + \Delta\sigma_{QCD} = \sigma_0 + \Delta\sigma_{vir} + \Delta\sigma_{soft} + \Delta\sigma_{hard} = \sigma_0(1 + \delta_{QCD}), \quad (4)$$

where $\delta_{QCD} \equiv \Delta\sigma_{QCD}/\sigma_0$ is the relative QCD corrections at the order of $\mathcal{O}(\alpha_s)$.

III. NUMERICAL RESULTS AND DISCUSSIONS

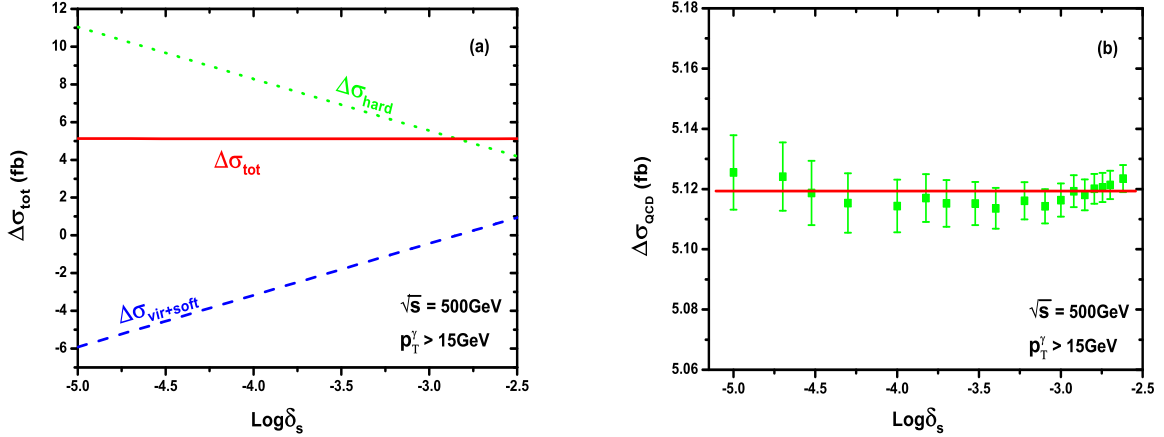


FIG. 1: (a) The dependence of the NLO QCD corrections on the soft cutoff δ_s in the process $e^+e^- \rightarrow t\bar{t}\gamma$ for $m_g = 10^{-8}$ GeV at $\sqrt{s} = 500\text{GeV}$; (b) The amplified curve marked with the calculation errors for $\Delta\sigma_{QCD}$ versus δ_s .

In the numerical evaluations, we take the input parameters of the SM as [30]

$$m_t = 171.2 \text{ GeV}, \quad m_e = 0.519991 \text{ MeV}, \quad m_Z = 91.19 \text{ GeV},$$

$$\sin^2 \theta_W = 0.2228, \quad \alpha(m_Z^2)^{-1} = 127.918 \quad (5)$$

Since the total cross section is independent of the non-physical parameters m_g and the soft cutoff δ_s ($\delta_s \equiv \Delta E_g/E_b$, $E_b = \sqrt{s}/2$), we display the curves of the NLO QCD corrections versus the cutoff δ_s for $m_g = 10^{-8}$ GeV in Fig.1(a), where we fix the renormalization scale $\mu = \mu_0 = m_t$. For the strong coupling constant $\alpha_s(\mu)$, we use the two-loop evolution of it with the QCD parameter $\Lambda^{n_f=5} = 226$ MeV and get $\alpha_s(\mu_0) = 0.1078$. It can be seen that the values of $\Delta\sigma_{hard}$ and $\Delta\sigma_{vir} + \Delta\sigma_{soft}$ vary with the change of the soft cutoff δ_s , but the total NLO QCD correction $\Delta\sigma_{QCD}$ is not dependent on δ_s within the reasonable calculation errors. In order to demonstrate this more clearly, we amplify the curve of $\Delta\sigma_{QCD}$ in Fig.1(b). We also verify that the total correction is indeed independent on m_g for the fixed δ_s . Therefore, in the following calculations, we take $\delta_s = 2 \times 10^{-3}$ and $m_g = 10^{-8}$ GeV.

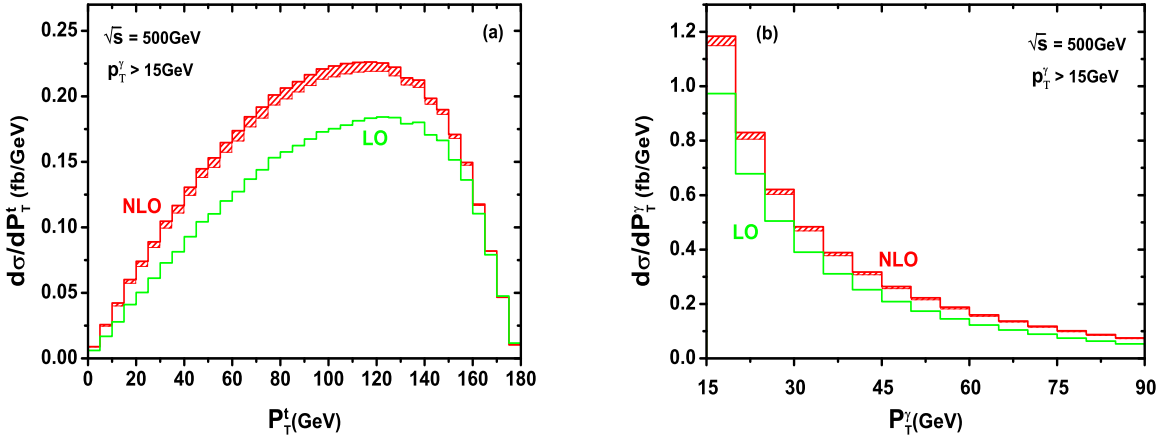


FIG. 2: The transverse momentum distributions of the top quark and photon at LO and NLO QCD respectively for the process $e^+e^- \rightarrow t\bar{t}\gamma$. The bands correspond to the variation of the renormalization scale in the interval $\mu_0/2 < \mu < 2\mu_0$.

In Fig.2(a-b), we show the transverse momentum distributions of the top quark and photon at LO and NLO QCD for $\sqrt{s} = 500$ GeV. In the calculations, we maintain the electron mass and impose a transverse momentum cut $p_T^\gamma > 15$ GeV to exclude soft photon emission. It can be seen that the QCD corrections greatly enhance the magnitudes of the LO differential cross section $d\sigma_0/dp_T$. But the shapes of these distributions are not dramatically changed. Most of the top quarks are produced in the region of $40 \text{ GeV} < p_T^t < 150 \text{ GeV}$; while the photons are inclined to distribute in the region $15 \text{ GeV} < p_T^\gamma < 60 \text{ GeV}$. We note that the detection of energetic photons produced by hard scattering goes through the

| p_T^γ (GeV) | $\sigma_0(fb)$ | $\sigma_{tot}(fb)$ | K |
|--------------------|----------------|--------------------|----------|
| 10 | 29.37(3) | 35.96(3) | 1.224(1) |
| 15 | 21.62(2) | 26.76(2) | 1.238(1) |
| 20 | 16.76(2) | 20.94(2) | 1.249(2) |
| 25 | 13.37(1) | 16.87(1) | 1.262(1) |
| 30 | 10.84(1) | 13.81(1) | 1.274(1) |

TABLE I: The LO cross sections, the NLO QCD total cross sections and K -factors under different p_T^γ cuts for process $e^+e^- \rightarrow t\bar{t}\gamma$ at $\sqrt{s} = 500\text{GeV}$.

definition of an isolation criterion [31]. However, this relies on the detailed Monte Carlo simulation, such as parton shower, which is beyond the scope of our study. In table I, we present the effects of different p_T^γ cuts on the LO cross section, the NLO QCD total cross sections and the K -factor of the $t\bar{t}\gamma$ production at $\sqrt{s} = 500\text{ GeV}$. For a higher p_T^γ cut, the cross sections become smaller and the relative corrections get larger.

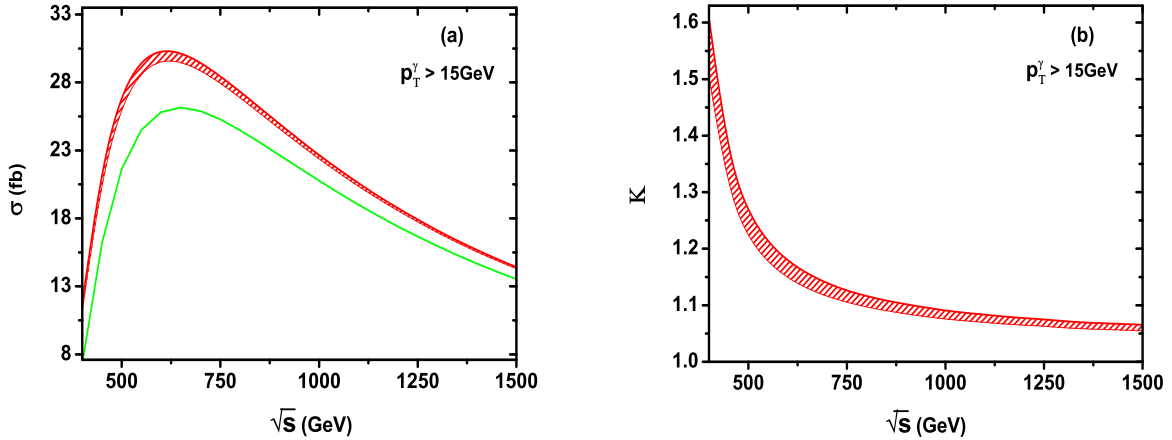


FIG. 3: (a) The cross sections of $e^+e^- \rightarrow t\bar{t}\gamma$ versus \sqrt{s} at LO and NLO QCD respectively; (b) The corresponding K -factor versus \sqrt{s} . The bands correspond to the variation of the renormalization scale in the interval $\mu_0/2 < \mu < 2\mu_0$.

In Fig.3(a-b), we give the dependence of cross sections and relative corrections on the center-of-mass energy \sqrt{s} . Since the process $e^+e^- \rightarrow t\bar{t}\gamma$ in the SM is induced by the pure electro-weak interaction at the order $\mathcal{O}(\alpha^3)$, the LO cross section will not be affected by the variation of the renormalization scale in the strong coupling. However, the NLO QCD

| $\sqrt{s} = 500 \text{ GeV}$ | | | | $\sqrt{s} = 800 \text{ GeV}$ | | | $\sqrt{s} = 1500 \text{ GeV}$ | | |
|------------------------------|--------------------|----------|-----------------------|------------------------------|----------|-----------------------|-------------------------------|----------|-----------------------|
| μ | $\sigma_{tot}(fb)$ | K | $A_{FB}^{t(tot)}(\%)$ | $\sigma_{tot}(fb)$ | K | $A_{FB}^{t(tot)}(\%)$ | $\sigma_{tot}(fb)$ | K | $A_{FB}^{t(tot)}(\%)$ |
| $\mu_0/2$ | 27.30(2) | 1.263(1) | 45.72(4) | 27.33(3) | 1.116(1) | 55.3(2) | 14.41(3) | 1.065(2) | 55.9(3) |
| μ_0 | 26.76(2) | 1.238(1) | 45.82(4) | 27.06(3) | 1.105(1) | 55.4(2) | 14.34(3) | 1.060(2) | 56.0(3) |
| $2\mu_0$ | 26.31(2) | 1.217(1) | 45.90(5) | 26.83(3) | 1.096(1) | 55.5(6) | 14.26(3) | 1.054(2) | 56.1(3) |

TABLE II: The numerical results of the NLO QCD total cross sections, the K -factor and the top quark forward-backward asymmetry at different values of the renormalization scale for the process $e^+e^- \rightarrow t\bar{t}\gamma$.

corrections are leading order in α_s and show a weak dependence on the scale, due to the suppression of the loop factor. We display the values of the NLO QCD total cross section (σ_{tot}), the K -factor and the top quark forward-backward asymmetry at the renormalization scale $\mu = \mu_0/2, \mu_0, 2\mu_0$ in table II. The uncertainty of the NLO scale dependence is approximately 3.7% (1.8%, 1.0%) for $\sqrt{s} = 500 \text{ GeV}$ (800 GeV, 1500 GeV) when the scale μ is varied between $\mu_0/2$ and $2\mu_0$. The uncertainty is defined as $\delta = [|\sigma(\mu_0/2) - \sigma(\mu_0)| + |\sigma(2\mu_0) - \sigma(\mu_0)|]/\sigma(\mu_0)$. When setting the scale at μ_0 , we find that the largest production rates of $t\bar{t}\gamma$ will reach about 25.82 fb and 29.98 fb at LO and NLO QCD respectively around $\sqrt{s} = 600 \text{ GeV}$, where the threshold effect may dominate. The corresponding relative QCD correction can be 16.1%. When \sqrt{s} is greater than 600 GeV, the cross sections drop rapidly, due to the s -channel suppression.

We also investigate the rapidity-differential cross sections of the top (anti-top) quark in Fig.4(a-b) and find that the events of the top (anti-top) quark for $y > 0$ are more (less) than that for $y < 0$. This asymmetry is caused by the huge interference effect between the photon and Z^0 boson mediated in the process $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow t\bar{t}\gamma$ [32]. The QCD corrections enhance the LO distributions of the top (anti-top) quark but do not distort their shapes significantly. In order to present this asymmetry, we can define the top quark forward-backward asymmetry (A_{FB}^t) in the process $e^+e^- \rightarrow t\bar{t}\gamma$ as

$$A_{FB}^t = \frac{N(y_t > 0) - N(y_t < 0)}{N(y_t > 0) + N(y_t < 0)}. \quad (6)$$

Here $N(y_t > 0)$ and $N(y_t < 0)$ denote the events of top quarks moving along or against a given direction, which is chosen as the direction of the incoming particle e^- in our calcula-

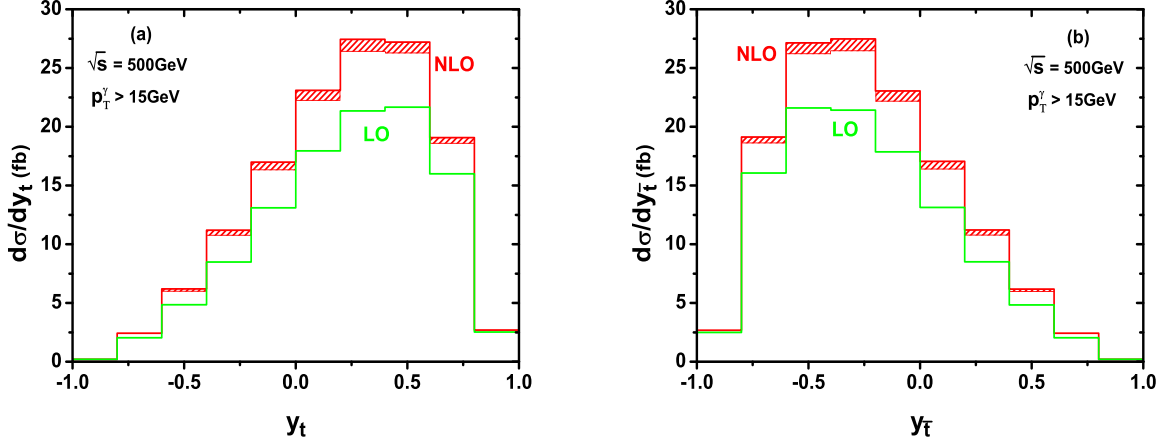


FIG. 4: The rapidity distributions of top and anti-top at LO and NLO QCD respectively in the process $e^+e^- \rightarrow t\bar{t}\gamma$. The bands correspond to the variation of the renormalization scale in the interval $\mu_0/2 < \mu < 2\mu_0$.

tions. In Fig.5, we can see that the QCD corrections give arise to a negative contribution to the LO forward-backward asymmetry. We list the values of A_{FB}^t at different scales in table.II. We find that the dependence of A_{FB}^t on the renormalization scale is very weak, due to the cancelation of strong coupling between numerator and denominator in Eq.(6). It is also noted that the value of this asymmetry is not sensitive to the collision energy when \sqrt{s} is greater than 900 GeV. The maximal values of A_{FB}^t can reach 58.1% and 56.4% for $\mu = \mu_0$ at LO and NLO QCD respectively.

IV. CONCLUSIONS

In this paper, we discussed in detail the one-loop QCD corrections to the process $e^+e^- \rightarrow t\bar{t}\gamma$ at the ILC. We found that the QCD corrections can significantly enhance the production rate of $t\bar{t}\gamma$ and show a weak dependence on the renormalization scale. The shapes of differential distributions of the top quark and photon are not be greatly affected by the QCD corrections. When fixing $\mu = \mu_0$, we found that the total cross section and the top quark forward-backward asymmetry can respectively reach 26.76 (27.06, 14.34) fb and 45.82 (55.4, 56.0)% at NLO QCD for $\sqrt{s} = 500$ (800, 1500) GeV.

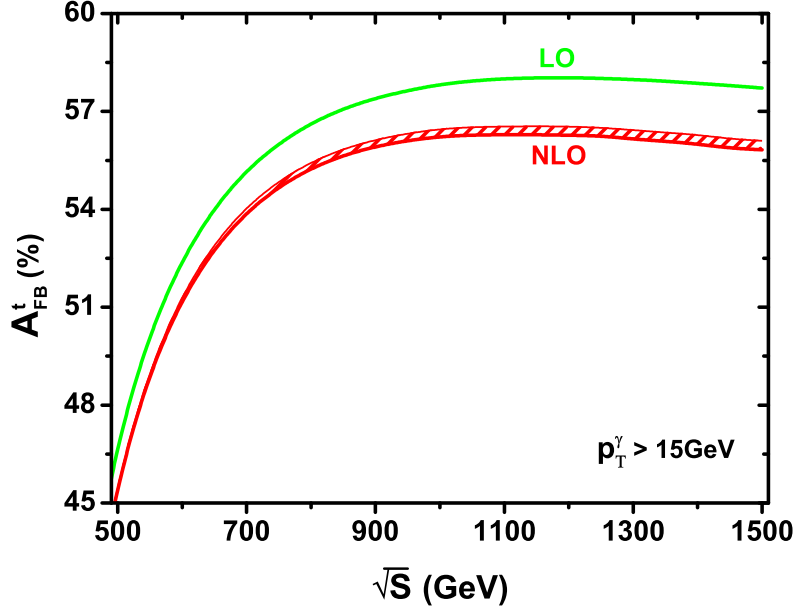


FIG. 5: The dependence of the forward-backward asymmetry A_{FB}^t on \sqrt{s} in the production of $t\bar{t}\gamma$ at LO and NLO QCD respectively at the ILC. The band corresponds to the variation of the renormalization scale in the interval $\mu_0/2 < \mu < 2\mu_0$.

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